Kinematic internal forces in deep foundations with inclined piles *

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Abstract

This paper presents a parametric study that looks into the influence of pile rake angle on the kinematic internal forces of deep foundations with inclined piles. Envelopes of maximum kinematic bending moments, shear forces and axial loads are presented along single inclined piles and 2×2 symmetrical square pile groups with inclined elements subjected to an earthquake generated by vertically-incident shear waves. Inclination angles from 0° to 30° are considered, and three different pile–soil stiffness ratios are studied. Amplification factors for the kinematic bending moments with respect to the vertical configuration are also presented. These results are obtained through a frequency–domain analysis using a boundary element – finite element code in which the soil is modelled by the boundary element method as a homogeneous, viscoelastic, unbounded region, and the piles are modelled by finite elements as Euler-Bernouilli beams. The rotational kinematic response of the pile foundations is shown to be a key factor on the evolution of the kinematic internal forces along the foundations.

1 Introduction

The interest in the seismic response of structures founded on inclined pile foundations, and in the behaviour of the raked piles themselves, has been growing very significantly in the last decade, giving rise to a significant number of both numerical and experimental publications on the topic. The motivations behind this interest and the main practical aspects of the issue are well described, for instance, by Giannakou *et al.* [1].

One of the more important aspects of the problem is understanding how dynamic internal forces evolve when inclining the piles, so that they can be detailed properly and all possible beneficial properties of raked piles can be exploited, even in seismically active regions. Several numerical [2, 3, 4, 5, 6, 1, 7] and experimental [8, 9, 10] studies have already looked into the seismic response of raked piles reaching very interesting conclusions for several particular cases. These results seem to confirm that inclined piles might develop kinematic forces at the pile-cap connection larger than those observed in their vertical counterparts, but they also suggest that batter piles might be beneficial in terms of the total response.

There exist a need for further research in order to generate enough data so that general conclusions on the topic can be drawn. This paper aims at presenting the results of a parametric analysis studying the influence of rake angle on the kinematic internal forces of single inclined piles and 2×2 square symmetrical pile groups with inclined elements, subjected to an earthquake

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produced by vertically-incident shear waves. Three different pile-soil stiffness ratios are also considered. The results are presented as envelopes of maximum kinematic bending moments, shear forces and axial loads along the whole foundation. Amplification factors of the bending moments with respect to the vertical configuration are also presented, as well as the evolution of maximum kinematic bending moments at pile heads with rake angle. The rotational kinematic response of the pile foundations is shown to be a key factor on the evolution of the kinematic internal forces along the foundations.

2 Methodology

The kinematic internal forces presented in this work have been obtained by making use of a coupled boundary element – finite element methodology developed in previous works for the dynamic analysis of pile foundations in the frequency domain. This soil – pile interaction model was first presented in Padrón *et al.* [11], work where the model was validated for the computation of dynamic stiffness and damping functions for vertical piles. After applying the model to several other related problems [12, 13, 14], the formulation was generalized to inclined piles in Padrón *et al.* [15] and validated for and applied to the computation of impedance functions for raked piles. The problem of impedance functions of end-bearing inclined piles was later treated in Padrón *et al.* [16]. Recently, the model was also validated for and applied to the computation of kinematic interaction factors of deep foundations with inclined piles in Medina *et al.* [17].

The soil is modelled as a linear, elastic, zoned-homogeneous unbounded domain and discretized by the boundary element method, while piles are modelled as Euler-Bernoulli linear–elastic beams by the finite element method. Soil and pile are assumed to be perfectly bonded. Pile cap is considered perfectly rigid and not in contact with the soil. Six-noded triangular elements or nine-noded quadrangular quadratic elements are used to discretize the soil surface, while three-noded beam elements are used for the piles. One of the main advantages of the formulation lies in the fact that the interface between pile and soil does not need to be discretized. For the problem at hand, the boundary integral equation is written in terms of diffracted fields and only for the nodes in the ground surface and for the pile nodes as internal points. This set of equations, together with the FEM equations for the piles, leads to a non-singular system of linear equations after imposing compatibility, equilibrium and boundary conditions. Once the solution is known, and as part of the post-processing, internal forces can be computed at the extreme nodes of each finite element as

$$\mathbf{F}(\omega) = (\mathbf{K} - \omega^2\mathbf{M})\mathbf{u}(\omega) + \mathbf{Q}\mathbf{q}(\omega) \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices, ω is the circular frequency of the excitation, $\mathbf{u}(\omega)$ and $\mathbf{q}(\omega)$ are the vectors of displacements and pile–soil interaction forces along the piles (primary variables of the model), and $\mathbf{Q}$ is the matrix that transforms nodal interaction force components into equivalent nodal forces. More details on the formulation can be found in the references given above. Once the complex frequency response function $\mathbf{F}(\omega)$, corresponding to all internal forces at extreme nodes of all pile elements, is computed along the whole frequency range of interest for a vertically–incident S wave field that produces unitary horizontal free-field ground motion (u_{ff}), the standard frequency domain method approach [18] is used to obtain the evolution of the internal forces in the time domain $\mathbf{f}(t)$ for a certain input accelerogram. Such time functions are used to produce the envelopes presented in section 3.

3 Results

3.1 Geometrical parameters and problem definitions

Two different cases are studied: a single inclined pile and a square 2×2 symmetrical pile group with battered elements. In both cases, the foundation is embedded in a homogeneous half-space.

Rotation of the pile head is prevented for the single pile ($\varphi_g = 0$). Regarding the pile group, the pile cap is assumed to be massless, infinitely rigid, free to rotate and move horizontally and vertically, and not in contact with the soil. Fixed connection between piles and cap is considered. The main geometrical parameters of the pile foundations are illustrated in figure 1: piles diameter $d = 0.6\,\mathrm{m}$, piles length $L = 12\,\mathrm{m}$, and center-to-center pile head separation $s = 3\,\mathrm{m}$, which corresponds to the slenderness and center-to-center pile separation dimensionless ratios $L/d = 20$ and $s/d = 5$. Besides, four rake angles are studied: $\theta = 0°$ (vertical pile), 10°, 20° and 30°. Piles are assumed to be reinforced concrete elements of solid circular cross-section with Young's modulus $E_p = 3 \times 10^{10}\,\mathrm{Pa}$ and a density of $\rho_p = 2500\,\mathrm{kg/m^3}$. Soil and pile Poissons ratios are $\nu_s = 0.4$ and $\nu_p = 0.25$. Three different soils are considered for the half-space, with shear wave velocities $c_s = 350$, 250 and $110\,\mathrm{m/s}$, respectively, and a density of $\rho_s = 1750\,\mathrm{kg/m^3}$, which corresponds to a $\rho_s/\rho_p = 0.7$ soil-pile density ratio, and pile-soil Young's modulus ratios of $E_p/E_s = 50$, 100 and 500 (softest soil). Soil hysteretic damping coefficient is defined as $\beta = 5\%$ in all cases.

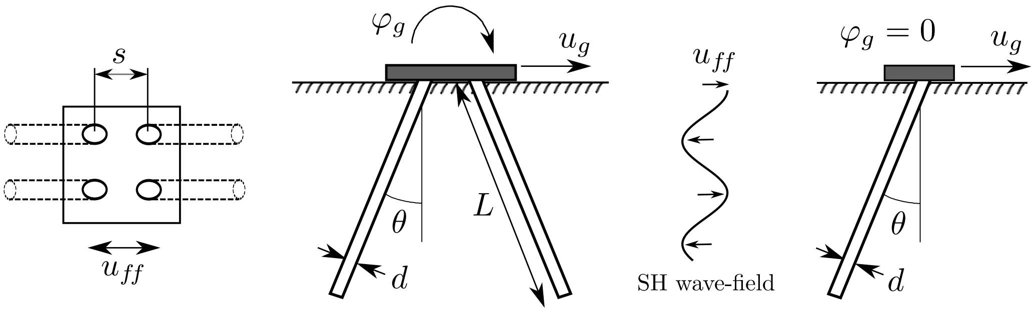

Figure 1: Problem description.

The seismic input motion is defined by a synthetic free-field ground surface horizontal accelerogram generated to be compatible with the type 1 elastic response spectrum for ground type C and 5% damping according to part 1 of Eurocode 8 [19]. Vertically–incident shear waves generating horizontal displacements parallel to the vertical plane containing the inclined piles (see figure 1) are assumed. The magnitude of the design ground motion in terms of reference peak ground acceleration is $a_g = 0.375\,\mathrm{g}$, while the duration of the signal is $t = 20\,\mathrm{s}$. Results have been obtained for three different synthetic accelerograms of these characteristics, being the conclusions of the study completely equivalent for all of them. For this reason, and for the sake of clarity and intelligibility of the plots, results are presented for one specific accelerogram. Stability and convergence analysis of the meshes have been performed in order to ensure the accuracy of the obtained results.

3.2 Comparison results

Now, in order to validate the methodology described in section 2 when used to compute internal forces in piles, envelopes of shear forces and bending moments along vertical piles are presented in this section against results obtained by the authors using a standard Beam-on-Dynamic-Winkler-Foundation formulation (BDWF). This BDWF, developed in the line of the works by Kavvadas and Gazetas [20] or Mylonakis [21], assumes a vertical pile embedded in a homogeneous, isotropic and linearly elastic half-space, with perfect bonding between soil and pile. Pertinent values of springs and dashpots are needed in its definition. References with suggestions about such values are abundant. In this respect, two groups could be made: *a*) references providing values tuned by comparison against results from more rigorous models (see for instance Kavvadas and Gazetas [20] or Makris [22]; or *b*) references providing values obtained directly from theoretical models as, for instance, the Novak-Baranov plane-strain elastodynamic theory. This is the case of the expressions proposed by Novak *et al.* [23]. An interesting revision regarding this topic can be found in Mylonakis [21] and Mylonakis [24]. In this paper, the coefficients proposed by Novak *et al.* [23] are used.

Figure 2 presents the envelopes of bending moments and shear forces along the vertical single pile defined above. The results obtained from the BEM–FEM coupling model are shown together with the results from the BDWF formulation described above. The dots on the curves corresponding to the BEM–FEM model represent the nodal values actually computed. The results are presented for the three pile-soil Young's modulus ratios that where defined above and that are going to be used in the next section. In general, the results obtained by both methodologies are in good agreement with each other. The envelopes of bending moments are exceptionally close taking into account the inherent differences between the two models. Along the pile, discrepancies both in bending moments and shear forces tend to grow for increasing contrast between soil and pile Young's moduli, with the BEM–FEM methodology yielding sligthly lower values in most instances, with the exception of the bending moments around the tip, where the BEM–FEM formulation yields larger values. In terms of bending moments for all stiffness ratios, and shear forces for $E_p/E_s = 50$, the most significant differences appear mostly near the tip of the pile, where the three–dimensional nature of the problem is inherently considered by the BEM-FEM model, which will also be able to include pile inclination in a rigorous manner.

3.3 Kinematic internal forces of deep foundations with inclined piles

This section presents results that allow to evaluate the influence of rake angle on the kinematic internal forces developed along the inclined piles subjected to vertically-incident S waves. The configurations under study are those defined in section 3.1. Figures 3, 6 and 7 present envelopes of kinematic bending moments, shear forces and axial forces, respectively, corresponding to a single inclined pile and a 2×2 pile groups with battered elements. First, second and third columns in figures 3 and 6 correspond to pile-soil stiffness ratios of $E_p/E_s = 50$, 100 and 500, respectively, while the first row corresponds to the single pile configuration and the second row corresponds to the 2×2 pile group. In case of figure 7, first and second columns correspond to axial compression and traction forces, respectively, for a single inclined pile, while third and fourth columns show the same information for a 2×2 pile group. First, second and third rows in figure 7 represent results corresponding to pile-soil stiffness ratios of $E_p/E_s = 50$, 100 and 500, respectively.

Figure 3 illustrates how the kinematic bending moments along the single pile decrease monotonically for increasing rake angles, irrespective of the pile-soil stiffness ratio. In the case of the pile group, the same trend is observed along the deepest two thirds of the foundations, while a completely different behaviour is observed in the shallowest part of the piles. There, the bending moments increase significantly at the head of the piles in the group when the piles are inclined. These differences between the maximum bending moments corresponding to the two deep foundations under study are due to the different rotational behaviours of single pile and pile group with inclined elements. As shown by Medina *et al.* [17], rake angle has very little effect on the rotational kinematic interaction factors of a single pile, while its influence on the rotational kinematic interaction factors of a pile group is very significant in the configuration studied in this work (direction of shaking parallel to the vertical plane containing the inclined piles). In this case, the magnitude of the kinematic rotation at the pile cap increases significantly when inclining the piles, and becomes out of phase with the horizontal free-field ground motion. This evolution in the kinematic rotation, together with the kinematic restriction imposed by the rigid cap, induces the increase in bending moments at the pile head observed in figure 3 for the pile group. Remember that rotation is prevented at the head of the single pile configuration studied here.

In order to gain a better insight into the magnitudes of the amplification factors affecting the envelopes of kinematic bending moments when inclining the piles, figure 4 presents the ratio between the maximum kinematic moment in the inclined pile to the maximum kinematic moment in the vertical pile, both at the same depth. Envelopes of bending moments are multiplied by

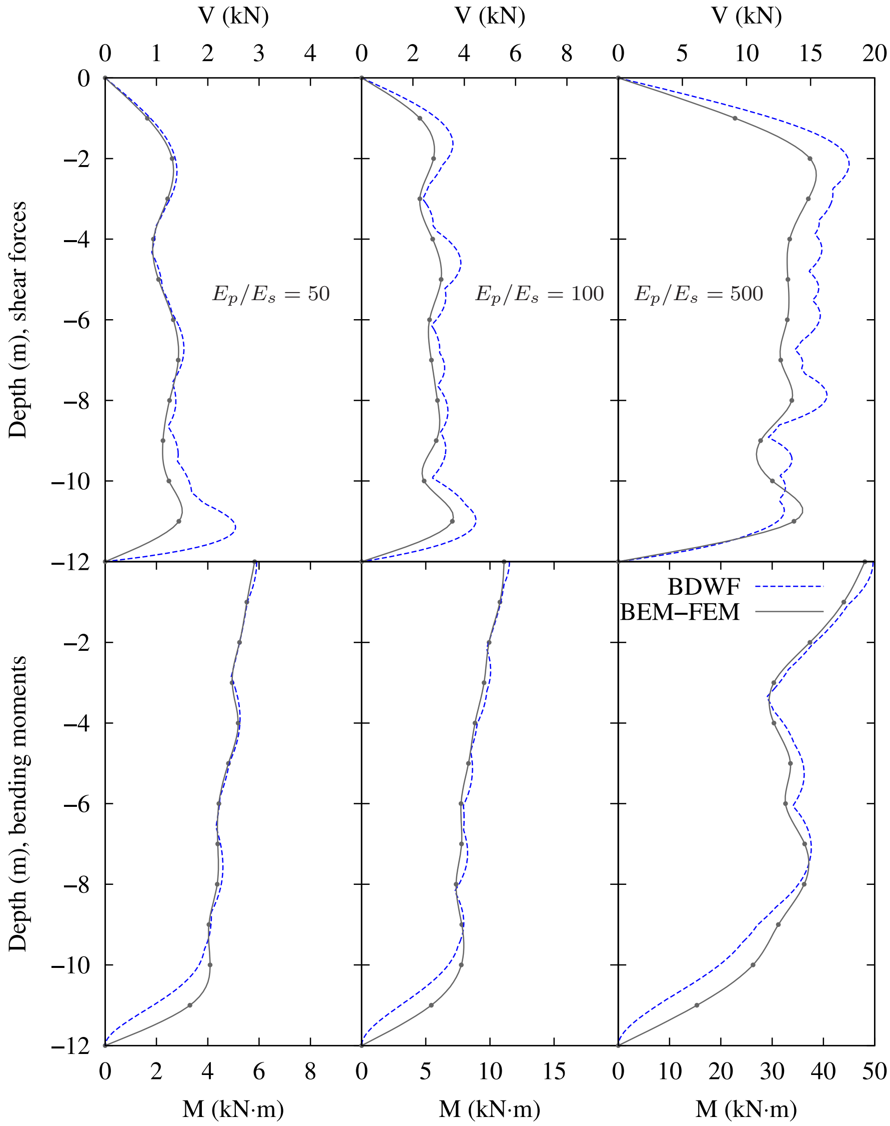

Figure 2: Envelopes of shear forces and kinematic bending moments corresponding to a single vertical pile in a homogeneous half-space submitted to vertically-incident shear waves. Comparison results.

factors below unity for the deepest parts of the piles in the group, and for all depths in the single pile, reaching reduction factors of up to 0.25 for 30°. At the head of piles in a group, on the contrary, bending moments are amplified by factors of up to 2.6 in some cases.

Both figures 3 and 4 show that, at pile heads, kinematic bending moments decrease monotonically with the rake angle in the case of single piles. In the case of piles in a group, on the contrary, the evolution is not monotonic. Such evolutions can be seen in figure 5 that, in order to analyse these trends, presents the maximum bending moments at the pile heads divided by the pile-soil stiffness ratio as a function of the batter angle. The differences between single pile and pile group configurations are well illustrated in this figure. The monotonic decrease in the case of single piles is completely different from the significant increase in the case of piles in a group, that show a maximum moment for angles around 20°. The largest variations are observed for the smallest soil–pile stiffness ratios, that is, for the stiffer soil.

Figure 6 shows that kinematic shear forces also decrease monotonically for increasing rake angles along the deepest parts of the piles. Along the shallowest parts, on the contrary, shear forces increase significantly with the batter angle in most cases both for single piles and for pile

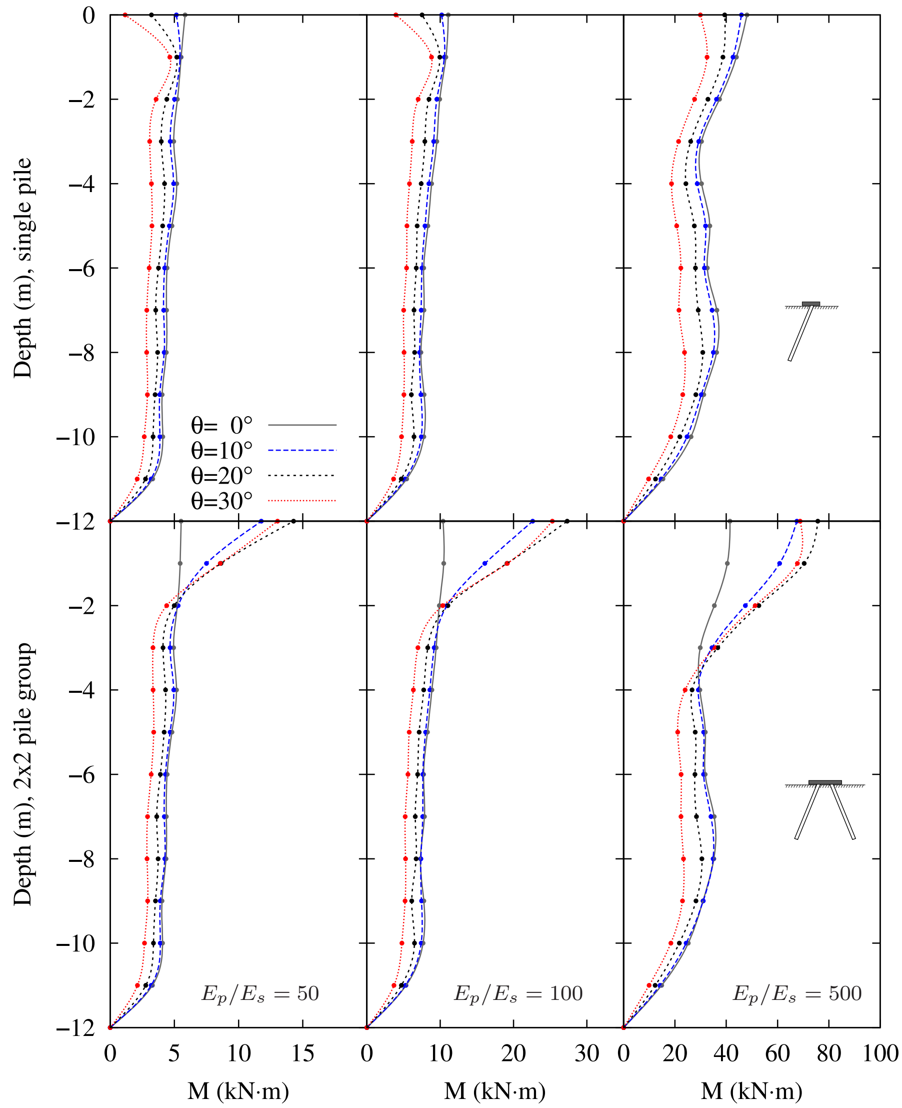

Figure 3: Envelopes of kinematic bending moments corresponding to a single inclined pile and a 2×2 pile group with battered elements submitted to vertically-incident SH waves. Pile-soil stiffness ratios considered: $E_p/E_s = 50$, 100 and 500

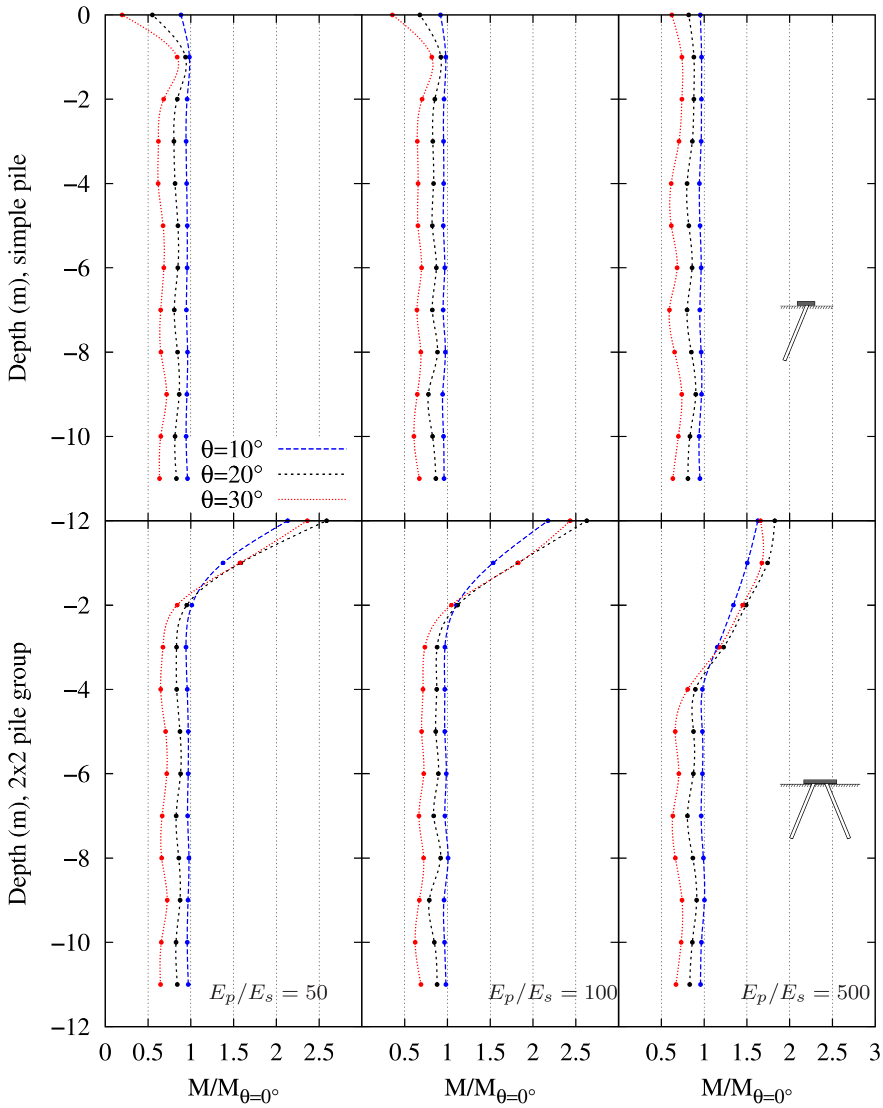

Figure 4: Ratios relating the envelopes of kinematic bending moments of inclined piles to those of vertical piles. Single piles and 2×2 pile groups submitted to vertically-incident SH waves.

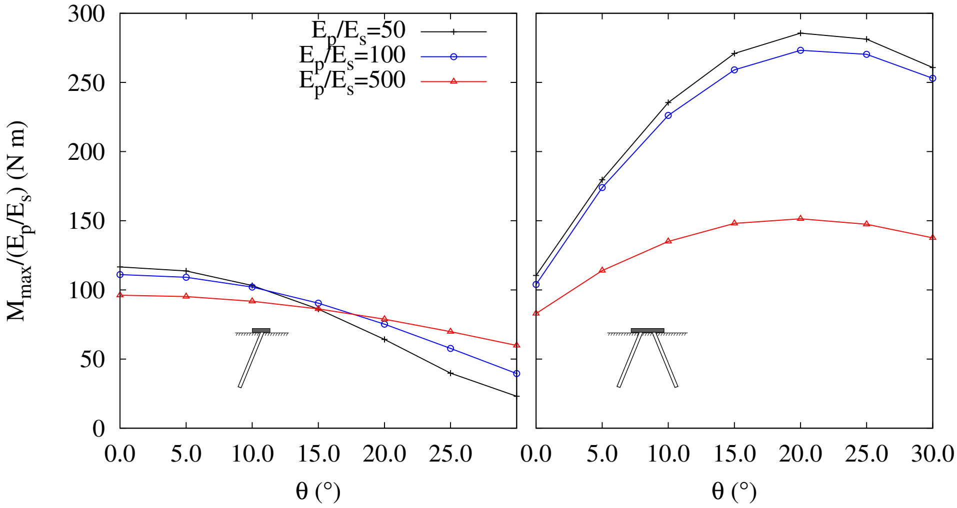

Figure 5: Maximum kinematic bending moment at pile head as a function of rake angle. Single piles and 2×2 pile groups submitted to vertically-incident SH waves.

groups, although the increases are much more important in the case of pile groups. In such configuration, as expected, shear forces at pile heads go from zero to very significant values but, in most cases, maximum values do not occur at pile heads but at depths of a few diameters. At such depths, the magnitude of the increase in shear forces observed when inclining the piles decreases when the pile–soil stiffness ratio increases, in such a way that, for $E_p/E_s = 500$, maximum shear forces correspond to the vertical case at all depths.

Finally, figure 7 presents the evolution of the axial forces along the piles. As expected, both compressive and tensile kinematic axial forces increase monotonically with the rake angle at all depths. The kinematic axial forces are null in the case of a single vertical pile. In the case of the pile group, they are not identically zero due to the kinematic restriction imposed by the pile cap. In all cases, the increase in the kinematic axial forces with the inclination of the piles is very significant, with maxima ocurring at depths depending on the pile–soil stiffness ratio and rake angle but always deeper than $L/2$.

4 Conclusions

The influence of pile rake angle on the kinematic internal forces along deep foundations with inclined elements is investigated in this paper making use of a time–harmonic linear–elastic boundary element – finite element coupling formulation in which the BEM is used to model the soil as a homogeneous viscoelastic half-space and the FEM to model the inclined piles as Euler-Bernoulli beams. Envelopes of maximum kinematic bending moments, shear forces and axial load, corresponding to different rake angles and soil properties, are presented for single inclined piles and for 2×2 pile groups with batter elements and subjected to vertically-incident SH waves. Fixed pile-cap connection is considered in the study. In the case of the single pile, rotation is assumed to be prevented at the pile head. The three main conclusions drawn from the analysis are:

- At depths distant from the pile heads, maximum kinematic bending moments and shear forces decrease always as the inclination angle increases, regardless of the foundation configuration and soil properties.
- The evolution of maximum kinematic bending moments and shear forces at pile heads depends on the configuration and boundary conditions at the pile cap. In the case of the

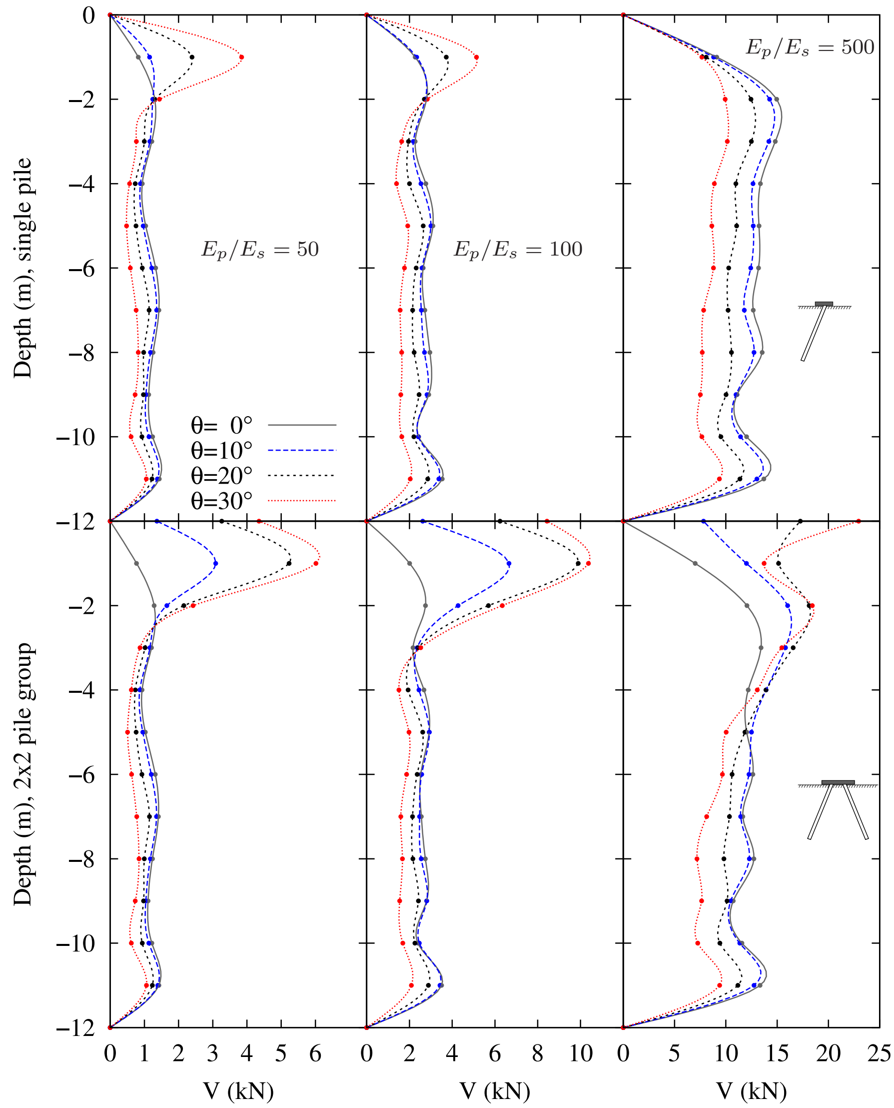

Figure 6: Envelopes of kinematic shear forces corresponding to a single inclined pile and a 2×2 pile group with battered elements submitted to vertically-incident SH waves. Pile-soil stiffness ratios considered: $E_p/E_s = 50$, 100 and 500

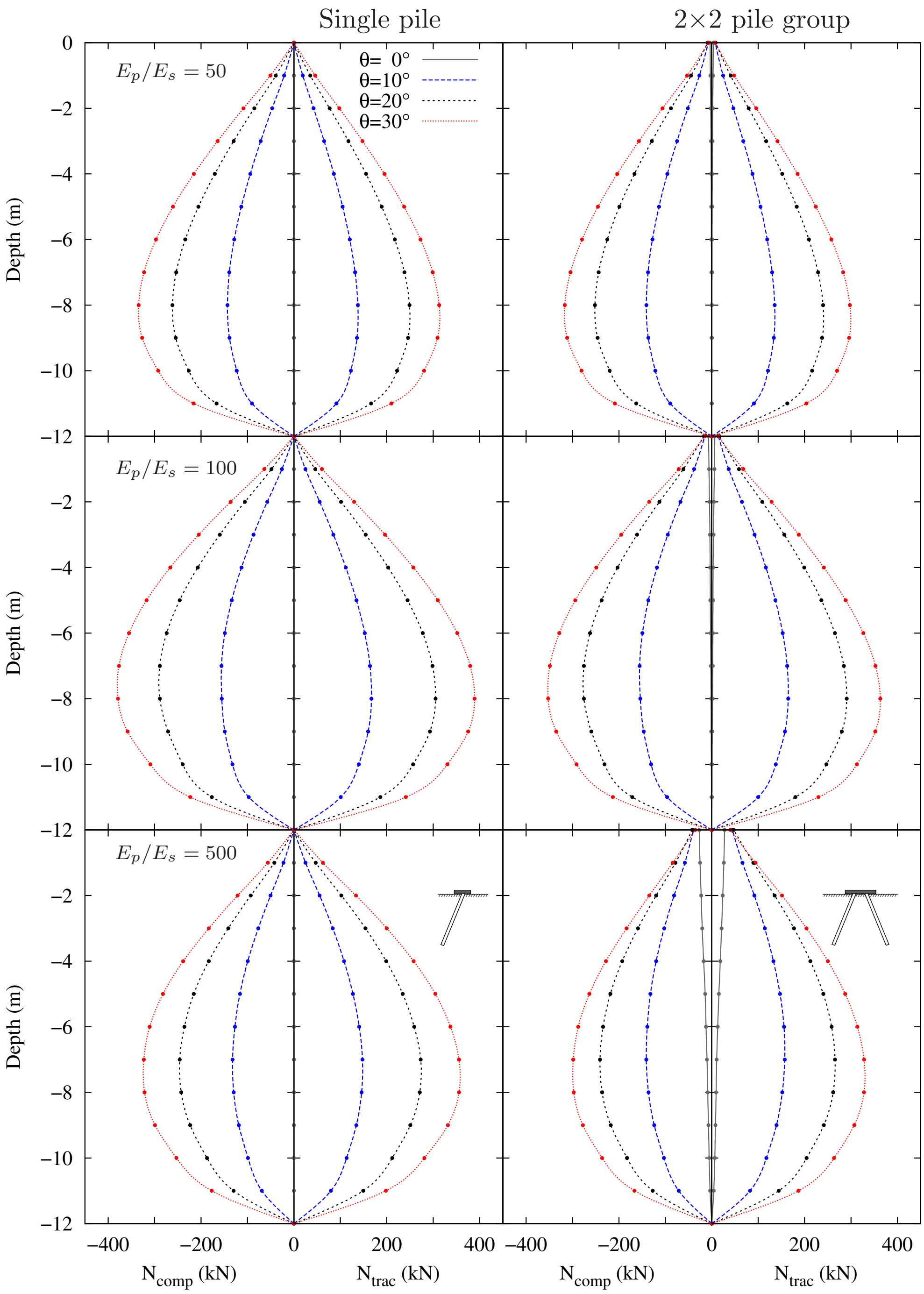

Figure 7: Envelopes of kinematic axial compression and traction forces corresponding to a single inclined pile and a 2×2 pile group with battered elements submitted to vertically-incident SH waves.

single inclined pile with rotation prevented at the pile cap, maximum kinematic bending moments at pile head decrease when inclining the piles, while shear forces tend to increase at shallow depths in most cases. In the case of the pile group, both maximum kinematic bending moments and shear forces tend to increase significantly around the pile head due to the kinematic restraint imposed by the pile cap together with the amplification of the rotational kinematic interaction factor for increasing rake angles. The kinematic bending moments at pile heads are found to be maximum for inclination angles around 20°, for which amplification factors with respect to the vertical configuration can reach values close to 3. Therefore, the rotational kinematic response of the foundation is a key factor when analysing the kinematic bending moments at the head of inclined piles: being able to model the actual rotational kinematic response, or assuming zero rotation at pile head, yields completely opposite results.

- Kinematic axial forces increase in all cases with pile inclination, both at pile heads and along all the pile buried length, which could lead to detrimental consequences such as an increased risk of buckling or a reduction in bending capacity [6].

The conclusions drawn here for the case of the pile group are in line with those reached in previous studies by Deng *et al.* [5], Giannakou *et al.* [4, 1] and Gerolymos *et al.* [6], and seem to confirm that pile inclination tend to produce larger kinematic internal forces around pile heads in comparison to vertical piles, although maximum kinematic bending moments and shear forces tend to decrease significantly along the deepest two thirds of the piles. However, there are several aspects of the topic that need further analysis:

- This study has focused only on fixed pile-cap connections, but other possibilities should be studied further. The assumption of hinged pile-to-cap connections modify the kinematic response of the pile, as shown by Ginnakou *et al.* [4], Gerolymos *et al.* [6] and Sadek and Shahrour [3]. As expected, the assumption of hinged pile-to-cap connections affects mainly the distribution of maximum bending moments.

- Indeed, non-linear phenomena could affect dynamic response of the inclined piles. Non-negligible residual bending moments in the case of seismic input, and significant local non-linearities around the pile head when the inclined piles where loaded from the top, where found in experimental studies by Escoffier [9] and Goit and Saitoh [25], respectively. In this respect, more comparison studies between numerical results and existing [8, 9, 10] or new experimental data, should be performed.

- Even though the use of inclined piles might seem to be detrimental in seismic areas in terms of their kinematic internal forces around the pile heads, there exist both numerical [2, 1, 7] and experimental [10] results that suggest that their use could be beneficial in many cases when looking at the total (kinematic plus inertial). This fact could be related to the role played by the rotational kinematic interaction response of the foundation observed here. Parametric studies looking into all variables of interest in these cases would be of great scientific significance.

Acknowledgments

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